Last class: $M\ddot{\vec{x}} + K\vec{x} = 0$

gives $\vec{x} = \bar{X}\cos\omega t$ ($\bar{X}$ ↑ constant)

$$-\omega^2\bar{X} + M^{-1}K\bar{X} = 0$$

$$\underbrace{M^{-1}K}_{A}\bar{X} = \underbrace{\omega^2}_{\lambda}\bar{X}$$

- standard e-vector, e-value problem

1.) can't be sure there are n e-vectors

2.) e-vectors not mutually perpendicular

<u>Slightly different approach</u>: change of coordinates before guessing solution:

$U = [\bar{X}_1 | \bar{X}_2 | \bar{X}_3 \ldots] \longrightarrow$ e-vectors of $M^{-1}K$

$\vec{X} = \bar{X}_1\eta_1(t) + \bar{X}_2\eta(t) + \ldots \longrightarrow \eta \rightarrow$ new coordinates

$\vec{H} = \begin{bmatrix}\eta_1(t)\\ \eta_2(t)\\ \ldots\end{bmatrix} \rightarrow$ so $\vec{x} = U\vec{H}$

Back to diff eq: $U\ddot{\vec{H}} + \underbrace{M^{-1}KU\vec{H}}_{U\Lambda\vec{H}} = \vec{0}$ $\qquad \Lambda = \begin{bmatrix}\lambda_1 & \cdots & \cdots \\ \vdots & \lambda_2 & \cdots \\ \cdots & \cdots & \lambda_3 \\ \cdots & \cdots & \cdots & \lambda_4\end{bmatrix}$

-multiply whole expression by U^{-1}: $\ddot{\vec{H}} + \Lambda\vec{H} = \vec{0}$

$$\begin{bmatrix}\ddot{\eta}_1 + \lambda_1\eta_1 = 0 \\ \ddot{\eta}_2 + \lambda_2\eta_2 = 0 \\ \ldots\ldots \\ \text{modal coordinates}\end{bmatrix}$$

↙ decoupled harmonic oscillator equations

aside: [V D] = eig(A)
>> A* V - V* D

$\begin{bmatrix}0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots\end{bmatrix}$

$\rightarrow \eta_1 = A_1\cos\omega_1 t + B_1\sin\omega_1 t$

$\eta = \ldots\ldots$

$\rightarrow \vec{x} = U\cdot\vec{H}$, $U = [\bar{X}_1 | \bar{X}_2 | \bar{X}_3 \ldots]$, $\vec{H} = \begin{bmatrix}\eta_1\\ \eta_2\\ \ldots\end{bmatrix}$

Official Normal Modes Method

$M\ddot{\vec{x}} + K\vec{x} = \vec{0}$ $\quad$ M is positive definite and symmetric

- there is some $\vec{q} = M^{1/2}\vec{x}$

$\quad \hookrightarrow \vec{x} = M^{-1/2}\vec{q}$

$\rightarrow M^{1/2}$ is symmetric, positive definite and exists

* a symmetric real matrix has real eigenvalues and eigenvectors, there are enough of them and they are orthogonal

Facts: A is symmetric $\rightarrow$ has n e-vectors, mutually orthogonal, λ_i are real

A is positive definite $\rightarrow$ $\lambda_i > 0$

$MM^{-1/2}\ddot{\vec{q}} + KM^{-1/2}\vec{q} = \vec{0}$ $\longrightarrow$ multiply both sides by $M^{-1/2}$

$M^{-1/2}MM^{-1/2}\ddot{\vec{q}} + M^{-1/2}KM^{-1/2}\vec{q} = \vec{0}$ $\longrightarrow M^{-1/2}MM^{-1/2} = I$

$\ddot{q} + \underbrace{M^{-1/2}K\vec{q}}_{\tilde{K}} = \vec{0}$ $\longrightarrow$ $\tilde{K}$ is symmetric, positive definite

Find e-values and e-vectors of $\tilde{K}$

$\rightarrow P = [\vec{q}_1 | \vec{q}_2 | \ldots]$ $\rightarrow$ eigenvectors of $\tilde{K}$ $\qquad \Lambda = \begin{bmatrix} \lambda_1 & \cdots & \\ \vdots & \lambda_2 & \\ & & \lambda_3 \ldots \end{bmatrix}$ $\rightarrow$ e-values of $\tilde{K}$

Another change of variables: $\vec{q} = P\vec{r}$ $\longrightarrow$ $\vec{r}$ = modal coordinates

($\vec{q}$ = e-vector of $\tilde{K} = M^{-1/2}KM^{-1/2}$ $\qquad = \vec{q}_1 r_1(t) + \vec{q}_2 r_2(t) + \vec{q}_3 r_3(t) + \ldots\ldots$

$\rightarrow P\ddot{\vec{r}} + \tilde{K}P\vec{r} = \vec{0}$

$\rightarrow P\ddot{\vec{r}} + P\Lambda\vec{r} = \vec{0}$ $\longrightarrow$ multiply by P^{-1}, $\Lambda \rightarrow$ diagonal

$\ddot{\vec{r}} + \Lambda \vec{r} = \vec{0} \rightarrow \begin{array}{l} \ddot{r}_1 + \lambda_1 r_1 = 0 \\ \ddot{r}_2 + \lambda_2 r_2 = 0 \end{array}$ } again decoupled harmonic oscillator equations

– gives $r(t)$'s

$\vec{X}(t) = M^{-1/2} \cdot \underbrace{P\,\vec{r}(t)}_{\vec{q}}$

$\vec{q}_1, \vec{q}_2 \ldots$ are mutually orthogonal